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Astronomical Implications of a Cosmological Theory of Light Propagation

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Abstract

It is shown that a simple cosmological model of light propagation permits the existence and observation of recession velocities greater than c, though the Einstein measure of these velocities remains less than c in accordance with relativistic theory.

The recent observation of quasars with red-shifts greater than unity is interpreted in terms of an exponential Doppler Law which is a consequence of the model. This law is further employed to provide an improved quantitative interpretation of the observed density of strong radio sources.

1. *Assumptions and Results*

Astronomical evidence suggests that the observable universe consists of a system of receding galaxies which appear to obey Hubble's Law; that is, that the relative velocity (as manifested by the Doppler effect) of a pair of typical galaxies (or fundamental observers) is proportional to the distance between them as manifested by their observed intensities corrected for the Doppler red-shift, etc.

It is natural to assume that the propagation of light, indeed of all forms of energy, will depend on the overall structure of our universe, and hence, that light travels in accordance with an hypothesis of McCrea (1962). This entails that a light-ray passes every fundamental observer (or typical galaxy) in its path with velocity c. The hypothesis is consistent with the equivalence of fundamental observers and also with the observation that light from distant sources has the same velocity as radiation originating locally.

It has been shown by the author (Prokhovnik, 1964, 1967) that this hypothesis, when applied to a uniformly expanding model of the universe, leads to a number of interesting results consistent with the principles and consequences of special relativity. In particular, it follows (Prokhovnik, 1964) that for a light-ray travelling between two fundamental observers F_1 and F_2, the epoch, t_1, of transmission by F_1 is related to the epoch, t_2, of reception by F_2, according to

$$\left.\begin{aligned} t_2 &= t_1 \exp(w/c) \\ \text{or} \qquad w &= c \log(t_2/t_1) \end{aligned}\right\} \tag{1.1}$$

where w is the mutual velocity of recession of F_1 and F_2 as deduced from the Doppler red-shift; and t_1, t_2 are measures of cosmic time whose present value is T, the reciprocal of Hubble's constant.

The relation (1.1) leads in turn to a relation between w and the corresponding Einstein measure, v, of the recession velocity, viz.

$$w = c \log \left(\frac{1 + v/c}{1 - v/c}\right)^{1/2} \tag{1.2}$$

This implies that no matter how large w is, v remains always less than c in accordance with the requirements of the usual relativistic transformations which are a consequence of the model. Hence the cosmological model as partly described by (1.1) and (1.2) places no restriction on the magnitude of w nor on the possibility of observing distant galaxies whose recession velocities have w-measures greater than c.

As a further consequence of (1.1), it is shown (Prokhovnik, 1967) that the red-shift ratio is related to w by the exponential law

$$\left.\begin{aligned} \frac{\Delta\lambda}{\lambda} &= \exp(w/c) - 1 \\ \text{or} \qquad w &= c \log(1 + z) \end{aligned}\right\} \tag{1.3}$$

on putting $\Delta\lambda/\lambda = z$.

Note that combining (1.2) and (1.3) yields

$$1 + z = \left(\frac{1 + v/c}{1 - v/c}\right)^{1/2} \tag{1.4}$$

which is the usual relativistic law in terms of Einstein's measure of velocity.

For a uniformly expanding universe the distance, r, between a pair of typical galaxies is related to w by

$$r = wT$$

where T is the reciprocal of Hubble's constant.

Hence also, invoking (1.3),

$$r = cT \log(1 + z) \tag{1.5}$$

which approximates the linear relation

$$r = cTz$$

if z is small compared to 1, but not otherwise. This result is consistent with the recent astronomical evidence presented by Hawkins (1962).

The mathematical results flowing from our cosmological model are closely similar to those of Milne's Kinematic Relativity (Milne, 1948). However, Milne's approach, in common with most others, lacks a specific mode of light propagation; hence in order to preserve the relativistic equivalence of fundamental observers, Milne introduced two scales of time related by a formula analogous to our (1.1). In our case the required equivalence is a direct result of the light hypothesis and it is the elaboration of this assumption which lends the model its interest and advantages.

2. *The Interpretation of Large Red-shifts*

The rapid development of radio-astronomical techniques has led to the detection of quasars with red-shifts numerically greater than unity. For instance, Bolton (1967) has recently reported the observation of an object labelled 0237-23 with a red-shift ratio of 2. Using the criteria proposed by McVittie (1965a, b), it is estimated that this object is 13,000 million light years distant, or slightly further if Milne's model is invoked. The latter is relevant since the appropriate Milne formula [equivalent to our (1.3)] is consistent with our own assumptions. In any case the distance estimated appears to be greater than what was previously considered to be the horizon of the universe, that is about 10,000 million light-years.

Now, in terms of the relation (1.3), a red-shift ratio of 2 corresponds to a value of $1{\cdot}1c$ for w which corresponds to $0{\cdot}8c$ for the Einstein measure v, using either (1.2) or (1.4). Thus the existence of recession velocities greater than c is seen to be consistent with our model and by no means contrary to the requirements of Special Relativity. It is suggested that a linear relation between the distance, r, of a galaxy and its recession velocity, w, may also obtain for w greater than c, though this of course implies again the logarithmic law (1.5) for the relation between r and z. More accurate data for the red-shifts and observed intensities of distant quasars will decide whether this conjecture is tenable; however, in terms of our model there is nothing theoretically inconceivable about it.

3. *The Density of Radio Sources*

The law (1.3) implies a more rapid diminution of a light-ray's frequency (and hence energy) with distance covered than is the case with a linear law. It has been shown (Prokhovnik, 1967) that invoking (1.3) instead of a linear Doppler law resolves Olber's paradox, even for a cosmological model of infinite dimensions.

Further, if the author's model (Prokhovnik, 1964) for the observation of radio sources is modified to incorporate the exponential law (1.3), then the usual relation, according to Bondi (1961), between the observed flux density, S, per unit bandwidth for a given frequency of reception, and the radio luminosity, P, of the source at the same frequency, is given by

$$S = \frac{P}{4\pi r^2(1+z)^2} = \frac{P}{4\pi c^2 T^2[\log(1+z)]^2(1+z)^2}$$

The expected number, dN, of radio sources in the range $r(pc)$ to $r+dr(pc)$, assuming that the occurrence of such sources depends on the square of the galactic density, may be taken as

$$dN = 4\pi r^2\, dr\, \rho_0 \frac{t_2^{\,6}}{t_1^{\,6}}$$

where $t_2 = T$, the present measure of cosmic time, and ρ_0 is the density of radio stars at the present time t_2. Hence using (1.1), (1.3) and (1.5),

$$dN = 4\pi c^3 T^3[\log(1+z)]^2 \rho_0(1+z)^5\, dz$$

The usual $N-S$ gradient, $G(z)$, is therefore given by

$$\frac{d(\log_{10} N)}{d(\log_{10} S)} = \frac{-54L^2}{(1+L)[18L^2 - 6L + 1 - (1+z)^{-6}]} = G(z)$$

where L denotes $\log(1+z)$.

The gradient, $G(z)$, for our modified model takes the values:

z	0·01	0·10	0·17	0·20	0·25	0·33	0·50
$G(z)$	−1·50	−1·57	−1·61	−1·63	−1·66	−1·71	−1·78

These values are somewhat lower than for the previous models given by the author (Prokhovnik, 1964, 1967), and so are in even better agreement with the concensus of recent radio observations.

Thus our model provides a quantitative interpretation of the observation by Ryle and others that the density of strong radio sources appears to increase with increasing distance of observation but ultimately converges.

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